SPECIFICATION OF MULTIPOLE TOLERANCES FOR THE APS QUADRUPOLE MAGNET

This note will address a proposed method for specifying the multipole tolerance for the design and production of APS quadrupole magnets. The tolerances for the multipole components for the quadrupole magnets will be set to that level which reduces the dynamic aperture by about 10-15% from the ideal machine dynamic aperture (as specified in CDR-87). This level may appear rather stringent, especially compared to the 50-60% reduction resulting from quad placement errors. However, when all tolerances are taken together, the residual dynamic aperture would be prohibitively small and commissioning would be difficult if these tolerances were at twice this level. The dynamic aperture was determined using the numerical tracking program RACETRACK.[1]

The multipole tolerances are defined by the expansion of the magnetic field given in Section II.4.2 of CDR-87. For quadrupoles a_0, $b_0 \equiv 0$ (quadrupole placement tolerances handle these dipole terms) and $b_1 \equiv 1.0 \text{ cm}^{-1}$ with $\frac{dB_y}{dx}\big|_{y=0} = B_0 b_1$ and $\frac{dB_y}{dy}\big|_{x=0} = -B_0 a_1$. Each of the 17 coefficients will have a tolerance value for the average value $(\bar{b}_n)$ of that coefficient over the ensemble of magnets (400 required) and for the spread or standard deviation (δb_n) which accounts for magnet-to-magnet variations. In the tracking program, the (δb_n) is taken to specify the rms spread of the magnet-to-magnet variations. The actual value for each b_n is assumed to be a random variable with a Gaussian distribution of mean $\bar{b}_n$ and standard deviation δb_n. No reordering or shuffling of magnets was used in computing the dynamic aperture, although this will be used to position magnets in the actual accelerator. The calculated dynamic aperture is the average of that obtained by generating eleven machines with different random sequences of magnet coefficients.

With 34 tolerances to be specified, the computational problem is undefined unless some relative scaling of the coefficients can be chosen. Although calculational techniques may be useful in setting levels for the allowed coefficients, the δb_n and non-allowed coefficients are difficult to compute in a way which will match the construction imperfections. In this case, we will use the relative magnitudes of these 34 tolerances taken from measurements of magnets built for two existing storage rings. Although different in design details from the APS quads, these two sets of magnets represent a very high-quality magnet (PEP insertion quad) and a lower-quality but more reasonable mass-produced magnet (FNAL pbar source quads).

The high-quality multipole coefficients we used were obtained from the summary of the PEP insertion quadrupoles quoted in the LBL 1-2 GeV Synchrotron Light Source CDR[2] but scaled to the same 4-cm aperture as the APS quads (see Appendix). The lower-quality multipole coefficients were obtained from measurements[3] of the FNAL SQA 500 series quadrupoles. The actual data for these measured integrated coefficients are shown in Table I, together with calculated[4] values for the APS quads with a non-optimized design for the quadrupole ends. The scaled a_n and b_n's assumed for the APS quadrupoles, with a 4-cm aperture radius, are given in Table II.

Figure 1 shows the effect on the dynamic aperture using the scaled coefficients from the PEP insertion quadrupoles. The data points and error bars represent the average and rms spread of the dynamic aperture for the 11 different sets of randomly distributed coefficients. Very little change from the ideal dynamic aperture is observed.

Figure 2 shows the effects of the more realistic FNAL quadrupole coefficients. The reduction of the dynamic aperture is about 10-15% and therefore represents the tolerance of the harmonic coefficients necessary for

Table I
Summary of FNAL SQA500 and PEP* Insertion Quadrupole Harmonic Measurements

Harmon n	FNAL(r_o=4.306 cm, a=4.445cm)		PEP(r_o=a=8cm.)
	$B(n)/B(1)$ x 10^{-4}	$A(n)/B(1)$ x 10^{-4}	$B(n)/B(1)$ x 10^{-4}
0	29.4±78.5	−4.71±43.7	----
1	0.996±.022x10^{4}	17.35±61.3	1.0
2	−0.77±2.08	−1.94±4.35	0.0±0.0128
3	−4.15±1.70	−0.35±0.49	0.0±0.853
4	−.164±0.36	0.32±0.42	0.0±0.000
5	−3.15±0.77	0.05±0.22	0.61±0.41
9	3.48±0.15	−0.07±0.14	−0.30±0.15

* Obtained from the ALS Conceptual Design Report, LBL (1986).

Table II
Scaled Harmonic Coefficients Used for Model of the APS Quadrupole Harmonic Coefficient Tolerances*

Harm n	#1 PEP insert. based	#2 FNAL SQ500 based	
	b_n x $[cm^{-(n)}]$	b_n $[cm^{-(n)}]$	a_n $[cm^{-(n)}]$
1	1.0	1.0	17.4±61.3x10^{-4}
2	0.0±5.0x10^{-6}	−1.9±5.4x10^{-5}	−5.0±11.2x10^{-5}
3	0.0±5.3x10^{-6}	−2.8±1.1x10^{-5}	−2.3±3.3 x10^{-6}
4	0.0±0.0	−2.8±6.2x10^{-7}	5.5±7.2 x10^{-7}
5	0.24±0.16x10^{-6}	−1.4±0.3x10^{-6}	0.2±1.0 x10^{-7}
9	−0.45±0.25x10^{-9}	6.9±0.3x10^{-9}	−1.4±2.8 x10^{-10}

*Scaled assuming the normalized coefficients are the same as for the measured magnets.

APS quads. However, the average value tolerances can be increased by 2-3 times without significantly reducing the dynamic aperture. The rms value of the harmonic coefficients dominates the effect on the dynamic aperture, but this could probably be improved by repositioning of the quads to suppress the dominant resonance terms. The algorithm for reordering will not be considered here, but will be discussed in a future note. Figure 3 shows the impact on the dynamic aperture from increasing the random coefficients by factors of 2 and 5 times the FNAL coefficients.

In the 1987 CDR, the tolerance on the harmonic coefficients was calculated in the same manner as described here, but only the duodecapole term (b_5) was considered (all other b_n, a_n = 0). In that case, the average value was limited to $|\bar{b}_5| \leqslant 5 \times 10^{-6}$ cm^{-4} taken to be positive or negative for all quadrupoles, regardless of the sign of the gradient. However, this tolerance can be increased by a factor of 20 or more if the sign of $\bar{b}_5$ depends on the sign of the gradient (i.e., magnitude of gradient increases or decreases for all quads). The tolerance on the spread was $\delta b_5 \leqslant 2.5 \times 10^{-6}$ cm^{-4} or a factor of 7.5 times the FNAL value specified here. This demonstrates the dependence of the tolerance specification on the number of coefficients considered and their relative scaling.

The APS quadrupole design is quite different from that of the PEP or FNAL magnets. Consequently, the multipole coefficients have been calculated[5] (in 2-D) in order to estimate the size of the average value ($\bar{b}_n$) that might result from production.* Table III shows these values compared to the scaled average values of the FNAL quadrupoles. All coefficients are quite similar in magnitude for these two sets of data, except for the b_4 term, which is almost

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*S. Kim demonstrated this scaling procedure using magnetic-field calculation programs with small changes in radial aperture.

Table III

Calculated Multipole Coefficients for APS Quadrupoles
Compared to the Scaled FNAL Average Values.

n	$b_n(\times\ 10^{-6}\ cm^{-n})$		$a_n(\times\ 10^{-6}\ cm^{-n})$	
	APS	FNAL	APS	FNAL
2	14	−19	75	−50
3	8.0	−28	0.0	−2.3
4	2.7	−0.28	0.0	0.55
5	6.3	−1.4	0.0	0.02
6	0.4	0.0	0.0	0.0
7	0.14	0.0	0.0	0.0
8	0.045	0.0	0.0	0.0
9	0.0	0.007	0.0	−0.0001

ten times the FNAL average value. The dynamic aperture has been calculated
using the calculated a_n, b_n for the APS design for $\bar{a}_n$, $\bar{b}_n$ and the FNAL
δb_n, δa_n. The resulting dynamic data differ only in the rms spread of dynamic
aperture being slightly smaller than that shown in Fig. 2. Increasing the
average values by factors of 2 and 5 times shows a small reduction of the
dynamic aperture, as presented in Fig. 4. This supports the idea that it is
the spread of the multipole coefficients which dominates the reduction of the
dynamic aperture.

In conclusion, the dynamic aperture reduction of 10-15% is expected if
the average value and spread of the multipole coefficients in the APS quads
are similar to those measured in the FNAL quadrupoles. This sets the
tolerance level for the APS quadrupole according to the CDR-87 standard. The

dominant terms, in terms of their impact on the dynamic aperture, are the δb_n, δa_n. Larger values for these terms may be acceptable but could limit the aperture with quad alignment errors, unless an effective magnet positioning algorithm is developed. Larger average values, up to two or three times the FNAL $\bar{b}_n$, $\bar{a}_n$, may be acceptable if the δb_n, δa_n don't also increase.

REFERENCES

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2. A. Avery et al., "1-2 GeV Synchrotron Light Source," LBL PUB-5172 Rev July 1986 and IEEE NS-26, p. 4033 (1979).

3. B. Brown, "Summary Sheets for SQA500 Measured Harmonic Components," private comm. (1988).

4. R. Lari, Argonne National Laboratory, private communication (March 1988).

5. S. Kim, Argonne National Laboratory, private communication (August 1988).

APPENDIX

The magnetic-field calculation programs and measurements with coils yield harmonic field components specified within a given coil radius (r_o) relative to that of the first harmonic. For quadrupoles these are defined as

$$\frac{B(n)}{B(1)}\Big|_{z=r_o} = \frac{\int B_o b_n r_o^{\,n} \, d\ell}{\int B_o b_1 r_o \, d\ell} = \frac{b_n}{b_1} r_o^{\,n-1}$$

and

$$\frac{A(n)}{B(1)}\Big|_{z=r_o} = \frac{\int B_o a_n r_o^{\,n} \, d\ell}{\int B_o b_1 r_o \, d\ell} = \frac{a_n}{b_1} r_o^{\,n-1}$$

where the b_n and a_n represent the effective coefficients over the entire quad.

In order to compare magnets with differing aperture radius a, the a_n and b_n will be scaled (best estimate for magnets with equal care in designing the pole tips[5]) as the percentage of the aperture that the measurement radius r_o represents. The normalized harmonic coefficients are defined as

$$B_y + iB_x = B_o \sum_{n=2} (b_n^a + i\, a_n^a) \left(\frac{z}{a}\right)^n \, .$$

The a_n^a and b_n^a coefficients will allow for a more straightforward comparison of the harmonic strengths of different magnets with different apertures. The relation between the coefficients used by RACETRACK are given by

$$a_n, \, b_n = \frac{a_n^a}{(a)^n}, \, \frac{b_n^a}{(a)^n} \, .$$

The values used in this dynamic aperture study are related to the measured values for a magnet with aperture radius a, by the relations

$$a_n(APS), \ b_n(APS) = \left(\frac{a}{a_{APS}}\right)^n \times a_n(\text{measured}), \ b_n(\text{measured})$$

with a_{APS} = 4 cm. This assumes that a^a_n, b^a_n are equal for the two magnets.

A final specification of the harmonic coefficients used by the programs DIMAT and MAD are the normalized gradient coefficients (for a_n = 0)

$$\bar{B} = B_\rho \sum_{m=0} \frac{K_m r^m}{m!}$$

and

$$\frac{B(n)}{B(1)} = \frac{K_n \ r_o^{n-1}}{K_1(n)!} = \frac{b^a_n}{b^a_1} \left(\frac{r_o}{a}\right)^{n-1}$$

or

$$\frac{b^a_n}{b^a_1} = \frac{K_n}{K_1} \frac{(a)^{n-1}}{(n)!} \ .$$

The measured values of the PEP insertion quadrupoles have been summarized by LBL in their CDR-86.[2] We have scaled these values for $a = r_o = 8$ cm to the APS-designed quadrupole magnets with a_{APS}=4 cm, as shown in Table II. The FNAL quadrupoles were measured with a coil of r_o=4.3 cm and have an aperture of a=4.45 cm. We have scaled their measured data to the APS aperture, as presented in Table II. No reduction of random coefficients has been included for measurement resolution of the harmonic coefficients. For some coefficients b_4, a_4, and a_5, the entire systematic and random coefficients could be due to the measurement resolution and finite statistics.
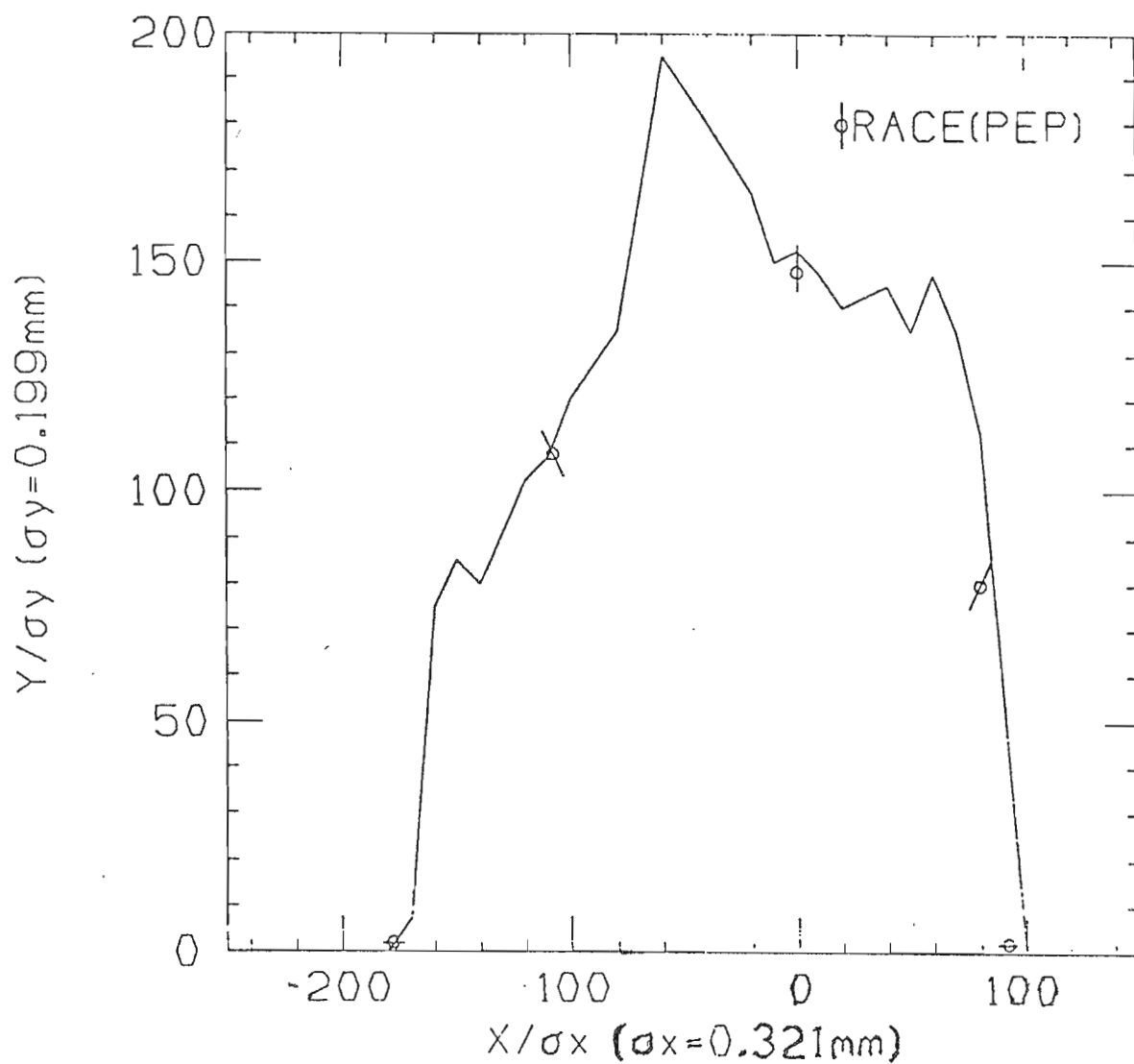
CG5B SCALED PEP HARMONICS

Figure 1

Dynamic aperture for the APS lattice with scaled PEP harmonic coefficients for the quadrupole magnets. The data points and error jbars represent the average and rms spread of the dynamic aperture for 11 random sequences of harmonic coefficients. The solid line represents the dynamic aperture for the ideal lattice and the dashed line the dynamic aperture with a 0.1-mm rms quadrupole alignment tolerance.
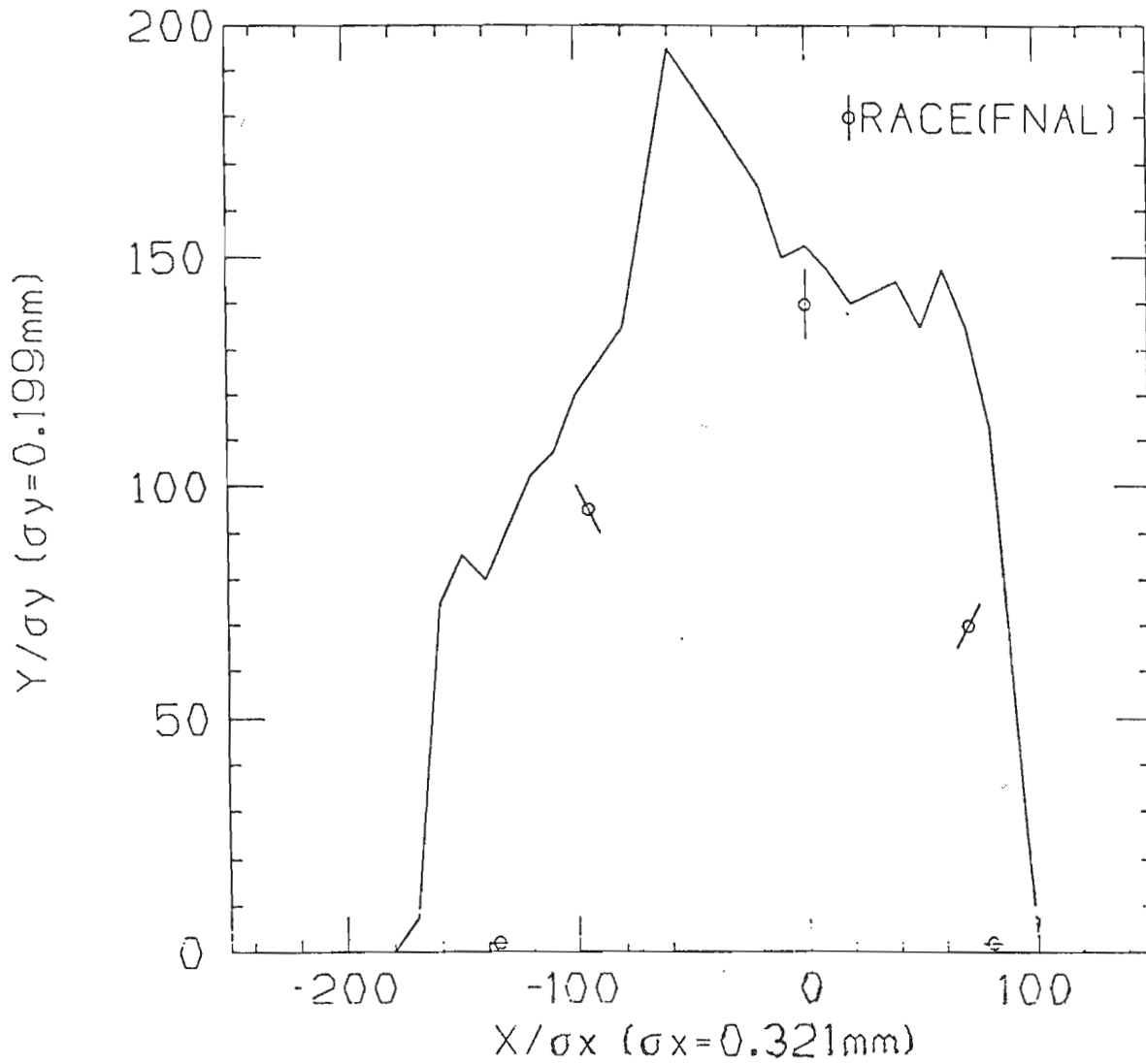
CG10B SCALED FNAL HARMONICS

Figure 2

Dynamic aperture for the APS lattice with scaled FNAL harmonic coefficients for the quadrupole magnets. The data points and error jbars represent the average and rms spread of the dynamic aperture for 11 random sequences of harmonic coefficients. The solid line represents the dynamic aperture for the ideal lattice.
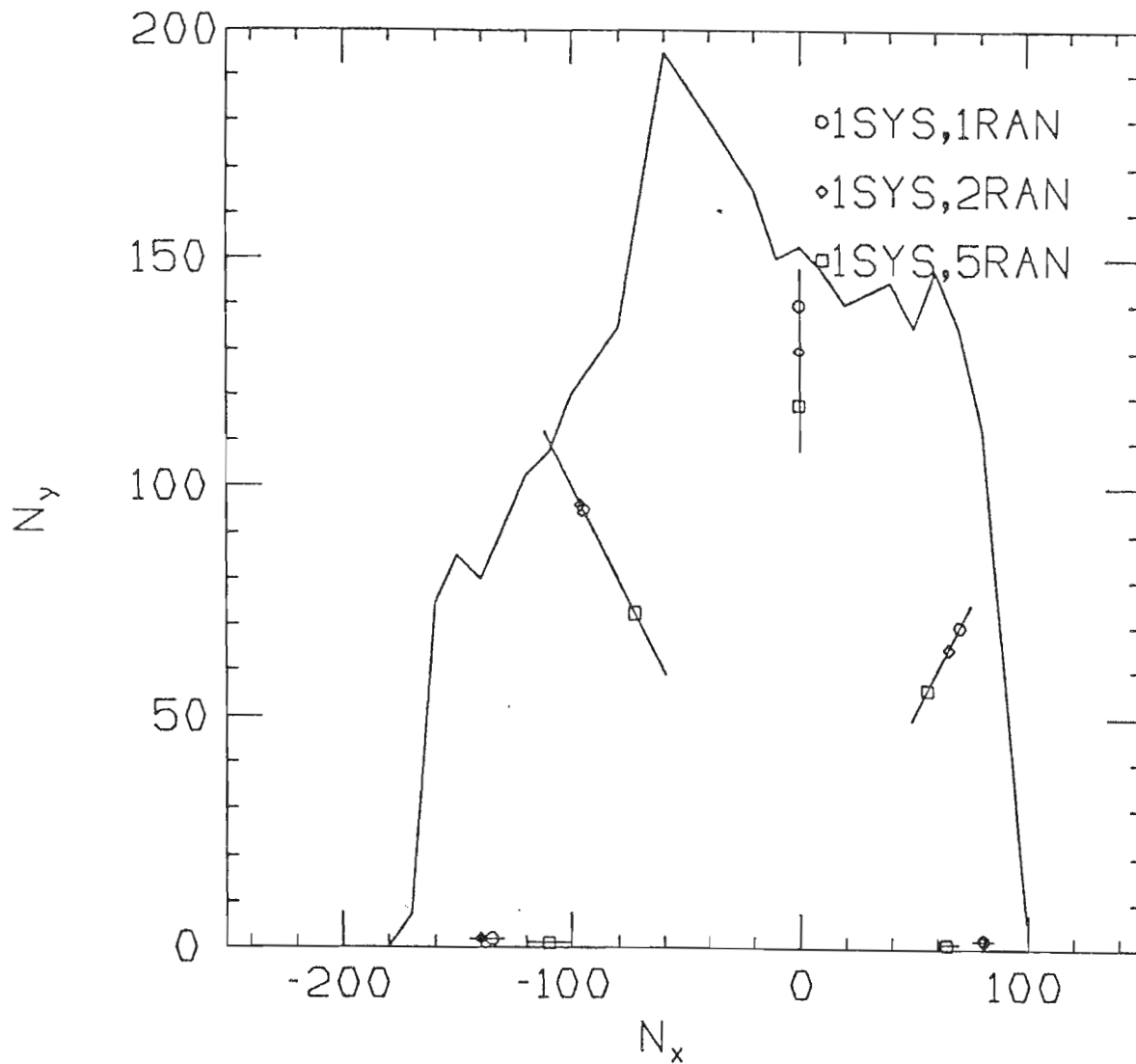
CG10B SCALED FNAL HARMONICS

Figure 3

Dynamic aperture for the APS lattice with scaled FNAL harmonic coefficients for the quadrupole magnets. The circles represent the FNAL coefficients as measured, the diamonds have the random coefficients increased by a factor of two, and the squares have the random coefficients increased by a factor of five.
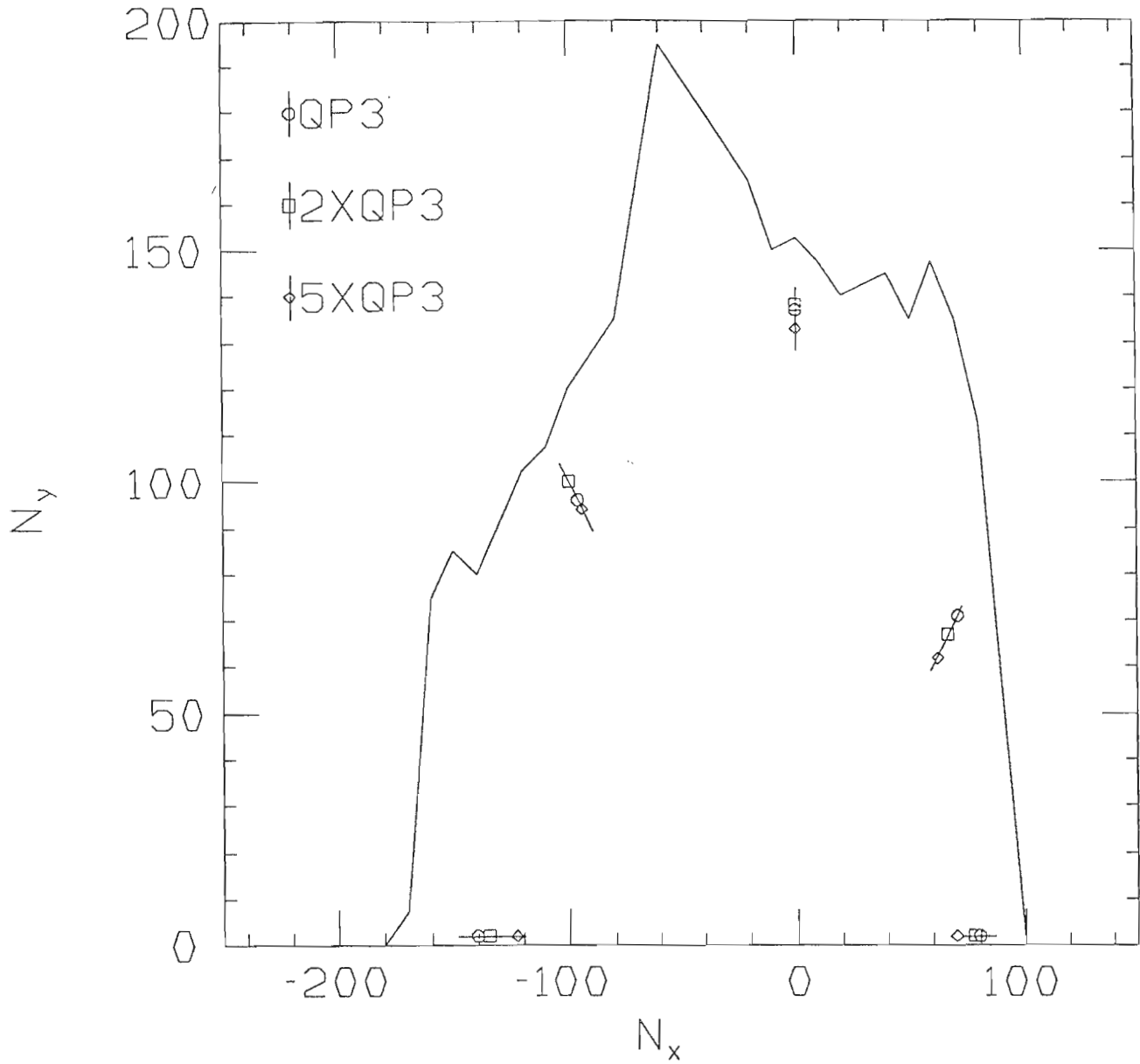
Figure 4

Dynamic aperture for APS lattice with multipole coefficients in the quadru-
pole given by: $\bar{b}_n$ from the QP3 quadrupole design calculations and δb_n from the
scaled FNAL measurements. The circles, squares, and diamonds represent one,
two, and five times the QP3 calculated coefficients, respectively.